The Evolution of Gender Norms, Division of Labor and Fertility

Seung-Yun Oh*

January 22, 2015

Abstract

Since 1990s a new pattern of positive correlation between fertility rates and female labor force participation emerged among developed countries. This recent trend seems inconsistent with conventional economic approaches that explain fertility decline as a result of the increasing opportunity costs of childrearing, predicting a negative correlation between fertility and women's labor force participation. To address the puzzle, I develop a model of the evolution of gender norms and fertility in various economic environments influenced by the level of women's wages. Randomly matched spouses make choices related to fertility regarding women's labor supply and the division of household labor based on their preferences shaped by gender norms. In the model, norm updating is influenced by both within-family payoffs and conformism payoffs from social interactions among the same sex. The model shows how changes in economic environments and the degree of conformism toward norms can alter fertility outcomes. When women's wages are very low, all men and women conform to traditional "separate spheres" gender norms and have high fertility. However, if women's wages are sufficiently high, women adopt an egalitarian "shared care" norm and engage in market work and men also adopt a "shared care" norm, thus couples achieve intermediate levels of fertility. However, the presence and strength of conformism modify these outcomes. If men strongly conform to traditional gender norms but women do not, then normative differences generate differences in preferences among spouses that lead to low fertility. The results suggest that the asymmetric evolution of gender norms between men and women could contribute to very low fertility, providing an explanation for the puzzle of the positive relation between fertility and women's labor force participation.

*Korea Insurance Research Institute

JEL Classification: C73, J1, D13, D19

Key words: gender norms, division of household labor, fertility, conformism, stochastic evolutionary game theory

1 Introduction

During the 20th century the total fertility rate declined substantially in many countries as women's labor force participation increased. In the final decades of that century, however, fertility trends stabilized in some countries, revealing a new pattern of positive correlation between fertility and female labor force participation.

Sweden, Norway, and the U.S., countries with relatively high levels of female participation in paid employment, also have relatively high fertility rates (at or slightly above replacement levels). By contrast, Japan, Spain, and Italy with relatively low levels of female participation, have relatively low fertility rates (well below replacement levels) (Feyrer et al., 2008). This recent trend seems inconsistent with conventional economic approaches (Mincer, 1963; Becker, 1965; Willis, 1973) that explain fertility decline as a result of the increasing opportunity costs of childrearing, predicting a negative correlation between fertility and women's labor force participation.

Fertility rates have declined particularly rapidly in Asia, from 5.3 children per woman in the late 1960s to 1.6 now. In Asian countries with the lowest marriage rates, the fertility rate is even lower, close to 1.0 (The Economist, Aug 2011). Below-replacement fertility results from both low marital fertility and non-marriage and/or delayed marriage. Strong traditional gender roles in those countries make it difficult for married women to both engage in paid employment and fulfill responsibilities for family care. Yet despite low levels of paid employment, married women seem reluctant to have large numbers of children.

In this paper I suggest that the asymmetric evolution of gender norms between men and women could contribute to very low fertility, providing an explanation for the positive relation between fertility and women's labor force participation. I consider two gender norms: a traditional "separate spheres" norm and an egalitarian "shared care" norm. The "separate spheres" norm dictates traditional gender roles in which men specialize in wage employment and the public sphere while women specialize in family care and the private sphere. The "shared care" norm allows women to engage in market work, and men and women share the cost of household labor and childrearing.

In a standard model of the marriage market, potential spouses find the best possible match. But this does not necessarily imply that they are able to find matches with spouses who share norms and preferences regarding the division of labor. Unlike other factors determining sorting in the marriage market - such as wealth, education, and outward appearance - potential partners may not be able to accurately observe one another's norms, which are easily misrepresented. In addition, if gender norms evolve asymmetrically between men and women - for example, most men conform to traditional gender norms, while most women

conform to more egalitarian norms – it could be hard to find a partner who has the same norms, so a person could decide to marry someone with different norms because it would be better than not marrying at all. Non-marriage can also be viewed as a possible outcome of the mismatched norm problem.

My approach differs from traditional economic theories of marriage in two ways. First, in contrast to Beckerian models, in which perfect specialization between men and women - corresponding to separate spheres norm - is considered efficient, I assume that marriage based on shared care norms can be more efficient in certain environments because diversification provides insurance against unemployment and strengthens paternal ties to children. Second, in contrast with the Coase Theorem, which assumes individuals can always implement efficient solutions through bargaining over redistribution, I emphasize bargaining rigidities that can lead to inefficient outcomes such as couples with mismatched preferences or a tendency to opt out of marriage.

I develop a model of the evolution of gender norms and fertility decisions in an asymmetric two-population game using evolutionary game theory. In the model, spouses are randomly matched regarding their norms. I refer to gender norms that have been "internalized" and have become in a sense a "preference", and define gender norms as informal governing rules that specify the division of labor: men decide whether to provide help on childrearing, while women decide whether to engage in market work. Fertility results from the choices that spouses make. If both husband and wife adhere to a separate spheres norm, then the wife will stay at home and tend to have high fertility. If both adhere to a shared care norm, the wife is likely to work and the husband provides help on childrearing. Her fertility will be lower than in the first case, due to the opportunity costs of childrearing, but intermediate levels will still be achieved. Among couples with mismatched norms, especially if husbands adhere to separate spheres norms but their wives have shared care norms, couples tend to have low fertility due to possible conflict over child care. Wives are likely to work whereas husbands do not provide help on childrearing. In this case wives will respond by lowering their fertility far below replacement levels because they are likely to suffer from "dual burden". If husbands adhere to shared care norms, but their wives adhere to separate sphere norms, conflict could also arise over the wives' foregone market income (the husbands may want wives to contribute household with market income, while they are willing to work fewer hours to be fathers). However, the conflict over child care, which is crucial in determining fertility, would be far less than the other mismatched couples.

In the model I consider women's market wage as an important economic factor that influences women's time allocation between market work and household work. Here women's market wage is considered as an expected wage which is determined by both women's wage

level and job opportunity because high wages do not necessarily go along with high female employment. Women who adhere to shared care norms will have higher payoffs in developed countries, where women's wages and opportunities for paid employment are sufficiently high, than in less developed countries.

Individuals have a tendency to adopt a particular behavior prevalent in the population (Boyd and Richerson, 1985; Bowles, 2004). Boyd and Richerson (1985) defines conformist transmission a tendency to copy the most frequent behavior in the population. Akerlof and Kranton (2000) argue that violating a society's behavioral prescriptions evoke utility loss such as anxiety and discomport. Similarly, formation of individual's gender norm is influenced by other agents' norms, especially those of the same sex. Most studies on norm evolution consider norms as an average behavior and the conformist payoff is modeled as utility gain depending on how close their action is to the social norm. Thus, the conformist payoffs depend on the frequency of agents adopting the behavior. However, specifying the frequency of norms is not enough to explain the transition of norms; the intensity of norms also matters. Even though the traditional "separate spheres" gender norm has weakened over time in many societies, including the U.S., it remains quite strong in East Asian countries, especially Japan and South Korea. Thus, I consider the degree of conformity, which captures the extent to which individuals in a society attach themselves to a norm. The greater a society's conformity toward existing norms, the greater will be the resistance to a transition to new norms.

Gender norms concerning division of labor are readily tested empirically. The International Social Survey Programme (ISSP) Family and changing gender roles survey includes specific questions on gender roles such as "A man's job is to earn money, a woman's job is to look after the home and family", and "What most women really want is a home and children". Analyzing such data, several recent studies attempt to explain international differences in fertility rates, investigating the relationships among gender inequality, the division of household labor, women's labor supply and fertility decisions. Using ISSP Family and changing gender roles survey, Laat and Sanz (2006) distinguish between gender attitudes within households and average attitudes of a society. They show that less egalitarian attitudes are associated with households with more children within countries; however, lower average fertility prevails in countries where households have less egalitarian views on average because the average attitude plays a role as social externality. Using the same data, Feyrer et al. (2008) also find a positive relation between men's household work and total fertility rate, and government family subsidies and fertility. Mills et al. (2008) compare gender equity and fertility decisions between Italy and Netherlands. They find that an unequal division of household labor is significantly associated with women's fertility intentions when those

women already suffer from a "dual burden" or "second shift". This literature, however, does not explicitly explore the effect of asymmetric gender norms on fertility decisions.

Evolutionary approaches to family have mainly dealt with issues such as the degree of altruism corresponding to biological relatedness (Hamilton, 1964); parent-offspring conflict (Trivers, 1974); sibling rivalry; gender differences in reproductive cost which implies a conflict over quality/quantity trade-offs between males and females (Folbre, 2006); and son preferences (Edlund, 1999). However, few studies pay attention to the evolution of gender norms. Iversen and Rosenbluth (2006) describes how different modes of production affect inter-gender bargaining power and the evolution of social norms, arguing that patriarchal social norms are the result of bargaining dynamics in labor-intensive agricultural societies. Some previous studies distinguished between patriarchal and egalitarian family contracts (Braunstein and Folbre, 2001; Geddes and Lueck, 2002; Folbre, 2006). This paper builds on existing studies by adding the insight that norms are internalized preferences that influence fertility decisions; providing a formal model of the evolution of gender norms, emphasizing in particular the role of conformism.

To examine the evolutionary process of changes in gender norms, I employ evolutionary game theory in which norm updating is determined partly by the within family payoffs based on each spouse's norm, and partly by the influence of social interactions among the same sex. First, I find evolutionary stable strategies of gender norms and corresponding fertility equilibrium in various economic environments. Then I examine how conformism alter the equilibrium. Second, I employ stochastic evolutionary model to study equilibrium selection in the long-run. In contrast to static and deterministic evolutionary games, stochastic games have an advantage in selecting equilibrium. The model extends existing stochastic evolutionary game theory in that it studies joint dynamics of between group and within group interactions for asymmetric two-population games.

Section 2 provides the main model. In section 3 I apply the result of the model to fertility decisions. In section 4 I study equilibrium selection in the long run. Concluding remarks follow.

2 The model

Suppose a society consists of N males and N females. For simplicity, I assume gender ratio to be sustained. There are two types of gender norms regarding the division of labor within family: a traditional separate spheres norm denoted by T and an egalitarian shared care norm by E. Each individual is endowed with a norm before marriage. When a male and a female are matched into a family, they choose strategies about how to divide household

work and market work. A husband has two choices whether to share childrearing work: {Not Help (NH) and Help (H)}. A wife has also two choices whether to engage in market work: {Not Work(NW) and Work(W)}. The choices are made corresponding to their norms: a husband with T norm will not help, while a husband with E norm will help; a wife with T norm will not work, while female with E norm will work.

Fertility (n) is determined by joint decision of husband and wife; it is a function of couple's time on childrearing. I assume that the cost of having childen is simply parents' time. Wives are endowed with time=1; h is time devoted on childrearing, so $1-h$ is time on market work. All husbands work outside. When a husband chooses to help on childrearing, he has to pay the cost of help denoted by $g > 0$, which can be regarded as leisure time for husbands. Let t_m and t_f be the time devoted to childrearing, then fertility function is given as $n(t_m, t_f)$, where n is increasing in both arguments.

I assume random matching and there are four possible outcomes. When both spouses have T norm, the wife will stay at home spending her entire time on childrearing, so they will have high fertility. When a male with T norm is matched by a female with E norm, the wife will engage in market work splitting her time between market work and childrearing, while the husband will not help on childrearing, so the fertility will be the lowest. When both spouses have E norm, the wife will work outside but the husband provide help on childrearing, so they will have the intermediate fertility. Finally, when a male with E norm is matched by a female with T norm, the wife will stay at home and the husband is willing to help, their fertility will be high. By letting $n_H := n(g, 1) = n(0, 1)$, $n_M := n(g, h)$, and $n_L := n(0, h)$, where $n_H > n_M > n_L$, the fertility results corresponding to four matching outcomes are given as

$$\begin{aligned}
(T,T) &\Leftrightarrow (NH,\ NW) : n_H \\
(E,T) &\Leftrightarrow (H, NW) \quad : n_H \\
(E,E) &\Leftrightarrow (H,\ NW) \quad : n_M \\
(T,E) &\Leftrightarrow (NH,\ W) \quad : n_L
\end{aligned}$$

Husbands' earning (I) is same to any family, I simply subtract husbands' income from all payoffs. Wive's earning can also be shared by husbands, but I disregard husbands' benefit from wives' wage because it does not alter husbands' choice; given wive's choice of work, husbands will benefit whatever choices they make. The utility of having children is simply

the number of children, n. The underlying payoffs of all outcomes are given as follows:

		Female	
		Not Work	Work
Male	Not Help	$(n_H,\ n_H)$	$(n_L,\ n_L + w)$
	Help	$(n_H - g,\ n_H)$	$(n_M - g,\ n_M + w)$

Without loss of generality, I assume that $n_H = 3n$, $n_M = 2n, n_L = n$. Thus, within family payoffs becomes

		Female	
		Not Work	Work
Male	Not Help	$(3n,\ 3n)$	$(n,\ n + w)$
	Help	$(3n - g,\ 3n)$	$(2n - g,\ 2n + w)$

(1)

Since having children is desirable and worth the cost for husbands, I assume $n > g$.

I denote the population fraction of T norm in a male population by x and the fraction of T norm in a female population by y. Let $s = (x, y)$ be the population state of norms. The set of all $s = (x, y)$ is

$$S = \{(x, y); x = \frac{i}{N}, y = \frac{j}{N},\ \text{for } i, j = 0, 1, ..., N\} \tag{2}$$

The utility (U) of an agent is the sum of two payoffs: a payoff from a family (π^W), and a payoff from social interactions (π^B). Let the within family payoff of player i with a norm k in a population state of (x, y) be $\pi^W(i, k, (x, y))$, where $i = m$ (male) or f (female), $k = T$ or E. Due to random matching, for example, a male with T norm will be matched by a female with T norm with probability y and a female with E norm with probability $1 - y$, thus his expected payoff will be $3ny + n(1 - y)$. Similarly the expected payoffs of other cases are as follows:

$$\begin{aligned}
\pi^W(m, T, (x, y)) &= 3ny + n(1 - y) \\
\pi^W(m, E, (x, y)) &= (3n - g)y + (2n - g)(1 - y) \\
\pi^W(f, T, (x, y)) &= 3nx + 3n(1 - x) = 3n \\
\pi^W(f, E, (x, y)) &= (n + w)x + (2n + w)(1 - x)
\end{aligned} \tag{3}$$

Agents derive utilities from social interactions by conforming their norms to others. Let the degree of conformism be σ. The conformist payoff of a norm will be higher as more people

adopt the norm, so it also depends on the population fraction of the norm. Let $\pi^B(i,k,s)$ be the conformism payoff of a player i with a norm k in a population state of $s=(x,y)$. Then

$$\begin{aligned}\pi^B(m,T,(x,y)) &= x\sigma,\ \pi^B(m,E,(x,y)) = (1-x)\sigma \\ \pi^B(f,T,(x,y)) &= y\sigma,\ \ \pi^B(f,E,(x,y)) = (1-y)\sigma\end{aligned}$$

The total payoff of an individual i with a k norm in a population state (x,y) is denoted by $U(i,k,(x,y)) = \pi^W(i,k,(x,y)) + \pi^B(i,k,(x,y))$. The total payoffs are;

$$\begin{aligned}U(m,T,(x,y)) &= 3ny + n(1-y) + x\sigma \\ U(m,E,(x,y)) &= (3n-g)y + (2n-g)(1-y) + (1-x)\sigma \\ U(f,T,(x,y)) &= 3n + y\sigma \\ U(f,E,(x,y)) &= (n+w)x + (2n+w)(1-x) + (1-y)\sigma\end{aligned} \tag{4}$$

3 Coevolution of norms and fertility

The primary goal is to study the effect of conformism on evolution of norms, consequently fertility decision. In this section I will study replicator dynamics to find asymptotically stable equilibrium. In a two-population and two-strategy game, an asymptotically stable equilibrium is also an Evolutionary Stable Strategy (ESS).

3.1 Equilibrium with no conformism

First, I study fertility and gender norm equilibrium with no conformism as a benchmark. From the payoff matrix in (1), it is easy to predict equilibrium. When women's wage is very low, staying at home will be dominant strategy for a wife whomever she is matched with. Given that wives stay at home, husbands are better off by not helping. Thus, everyone in the population adopts T norm. On the other hand, if women's wage is very high, more women will adopt an egalitarian norm and engage in market work, and husbands also adopt an egalitarian norm corresponding to an increase in population fraction of female with E norm. I will verify this intuition by solving replicator dynamics of the game.

The replicator equations for the game are given as;

$$\begin{aligned}\dot{x} &= x(1-x)\{\pi^W(m,T,(x,y)) - \pi^W(m,E,(x,y))\} \\ \dot{y} &= y(1-y)\{\pi^W(f,T,(x,y)) - \pi^W(f,E,(x,y))\}\end{aligned} \tag{5}$$

The stationary states for (5) are defined to be the state (x, y) where $\dot{x} = 0$ and $\dot{y} = 0$. Thus from $\pi^W(m, T, (x, y)) = \pi^W(m, E, (x, y))$ and $\pi^W(f, T, (x, y)) = \pi^W(f, E, (x, y))$, all stationary states are given by

$$(x^*, y^*) = (0, 0), (0, 1), (1, 0), (1, 1), (\frac{w-n}{n}, y), (x, \frac{n-g}{n})$$

It is easy to see that $\dot{x} > 0$ if $y > \frac{n-g}{n}$ and $\dot{y} > 0$ if $x > \frac{w-n}{n}$. Note that asymptotically stable equilibrium of the replicator dynamics differs depending on women's market wage, and this gives three wage regimes

$$(i)\ w < n, \quad (ii)\ n < w < 2n, \quad (iii)\ 2n < w \tag{6}$$

Since $n > g$, the value $\frac{n-g}{n}$ lies between 0 and 1. Thus the population fraction of male with T norm will increase if the population fraction of female with T norm is greater than $\frac{n-g}{n}$ and vice versa. This holds for all three cases. Regarding the evolution of y, each wage regime will induce different results. For the case (i), w is lower than n, the value $\frac{w-n}{n}$ is negative, then $x > \frac{w-n}{n} > 0$ and $\dot{y} > 0$ for all values of x. All the states converge to the state (1,1) in which everyone adopts T norm and families have high fertility. In case of (iii), w is higher than $2n$, the value $\frac{w-n}{n}$ is greater than 1, which implies $x < \frac{w-n}{n} < 1$ and $\dot{y} < 0$ for all values of x. The population fraction of female with T norm decreases. Thus the state (0,0), all adopt E norm, is the only asymptotically stable state for the case. The vector field diagrams for the three cases are given in Figure 1. Panel I, II, and III correspond to the case (i), (ii), and (iii).

$itbpF2.0954in1.6336in0inch2_f1_p1.bmpitbpF2.2528in1.638in0inch2_f1_p2.bmpitbpF1.9035in1.6397in0inch2_f$

Figure 1. Replicator dynamics and ESS without conformism

In case of (ii), there are two asymptotically stable equilibria and corresponding two ESS's. The value $\frac{w-n}{n}$ is between 0 and 1, thus y can increase or decrease contingent on the state of x; whether x is greater or less than $\frac{w-n}{n}$. "History matters" in this case because a population will move towards (1,1) or (0,0) depending on the initial state as shown in Panel II. Note that the mismatched norm cannot be an ESS if there is no conformism. Proposition 1 reports the result.

Proposition 1 *There are the following fertility regimes as a result of the evolution of gender norms.*

- *If women's wage is sufficiently low such that $w < n$, population state of all male and*

female adopting a separate spheres norm is an ESS and families have high fertility .

- *If women's wage is an intermediate level such that $n < w < 2n$, both all adopting a separate spheres norm and having high fertility and all adopting a shared care norm and having intermediate level fertility are ESSs.*

- *If women's wage is sufficiently high such that $2n < w$, population state of all male and female adopting a shared care norm is an ESS and families have intermediate level fertility.*

3.2 Equilibrium with conformism

As an extreme case, consider first that individuals have only conformist payoff. Then the replicator equations are given as;

$$\begin{aligned} \dot{x} &= x(1-x)\{\pi^B(m,T,(x,y)) - \pi^B(m,E,(x,y))\} \\ \dot{y} &= y(1-y)\{\pi^B(f,T,(x,y)) - \pi^B(f,E,(x,y))\} \end{aligned}$$

The stationary states are $(x^*, y^*) = (0,0), (0,1), (1,0), (1,1), (\frac{1}{2}, y), (x, \frac{1}{2})$. Then individuals simply adopt the norm which is prevalent (greater than $\frac{1}{2}$) in a society. The vector field diagram is given in Figure 2.

$itbpF1.9121in1.6613in0inch2_f2.bmp$

Figue 2. Replicator dynamics with conformist payoff only

All four states, (0,0), (0,1), (1,0) and (1,1), are asymptotically stable and ESS.

Now I consider both within family payoffs and conformist payoffs. Agents update their norms based on the sum of both payoffs. The replicator equations are given as

$$\begin{aligned} \dot{x} &= x(1-x)\{U(m,T,(x,y)) - U(m,E,(x,y))\} \\ \dot{y} &= y(1-y)\{U(f,T,(x,y)) - U(f,E,(x,y))\} \end{aligned} \tag{7}$$

By solving the third component of each equation, the critical values ensuring $\dot{x} = 0$ and $\dot{y} = 0$ are given by $x = \frac{1}{2} + \frac{n-g}{2\sigma} - \frac{n}{2\sigma}y$ and $y = \frac{1}{2} + \frac{w-n}{2\sigma} - \frac{n}{2\sigma}x$. Conformist payoffs tilt the solution trajectories. Now there is a possibility that the mismatched norm can be asymptotically

stable and ESS. Figure 3 shows the situation.

$$itbpF2.386in2.0626in0inch2_f3.bmp$$

Figure 3. Replicator dynamics with within family and conformist payoffs.

Our interest is to find out the condition when the state (1,0), all males with T norm and all females with E norm, is asymptotically stable and becomes an ESS. The shaded area in Figure 4 is the basin of attraction for the state (1,0). If a population state is in the shaded area, it will converge to the mismatched norm equilibrium (1,0). The stationary solution trajectory of $\dot{x} = 0$ includes the state $(x, y) = (\frac{1}{2} - \frac{n-g}{2\sigma}, 0)$ and $\dot{y} = 0$ includes the state $(1, \frac{1}{2} - \frac{2n-w}{2\sigma})$. The state (1,0) will have a basin of attraction when $x = \frac{1}{2} - \frac{n-g}{2\sigma}$ lies between 0 and 1 for $\dot{x} = 0$ trajectory; and $y = \frac{1}{2} - \frac{2n-w}{2\sigma}$ lies between 0 and 1 for $\dot{y} = 0$ trajectory. This leads to the following Proposition.

Proposition 2 *Suppose the degree of conformism is sufficiently high such that $\sigma > n - g$ and $\sigma > |2n - w|$. Then the state (1,0) is an ESS and it has basin of attraction, $\{(x, y)|\ x > \frac{1}{2} + \frac{n-g}{2\sigma} - \frac{n}{2\sigma}y$ and $y < \frac{1}{2} + \frac{w-n}{2\sigma} - \frac{n}{2\sigma}x\}$.*

Proof. *If $x > \frac{1}{2} + \frac{n-g}{2\sigma} - \frac{n}{2\sigma}y$ and $y < \frac{1}{2} + \frac{w-n}{2\sigma} - \frac{n}{2\sigma}x$, we have $\dot{x} > 0$ and $\dot{y} < 0$ from (7). Thus the population fraction of male with T norm and the population fraction of female with E norm increase, converging to the state (1,0).* ■

Now I explore how conformism changes a society's norm equilibrium and fertility. Consider the case (iii) wage is high, $w > 2n$, in (6). The state (0, 0), all males and females adopt E norm, is the only ESS without conformism. If the degree of conformism is high, $\sigma > n - g$, then the set of states satisfying $x > \frac{1}{2} + \frac{n-g}{2\sigma} - \frac{n}{2\sigma}y$ and $y < \frac{1}{2} + \frac{w-n}{2\sigma} - \frac{n}{2\sigma}x$ converges to the state (1,0), all males adopt T norm while all females adopt E norm, and families have very low fertility.

Panel I Panel II

$$itbpF1.9259in1.9259in0inch2_f4_p1.epsitbpF1.919in1.919in0inch2_f4_p2.eps$$

Without conformism With conformism

ESS:$(x, y) = (0, 0)$ ESS $(x, y) = (0, 0).(1, 0), (1, 1)$

Figure 4. Vector fields with and without conformism. Parameter values: $n = 5; g = 3; w = 11; \sigma = 4$

Figure 4 shows an example of the case (ii). The left hand side graph describes the vector

field of the replicator dynamics without conformism and the right hand side graph is the vector field with conformism. The Proposition 2 suggests that under conformism, there can be multiple ESSs even though unique ESS is obtained without conformism. This shows the role of conformism in explaining mismatched norms and low fertility equilibrium. Consider two societies with traditional norms; one with relatively low degree of conformism and the other with relatively high degree of conformism. Suppose women's market wage increased sufficiently high and considerable fraction of women already engaged in market work. In the society with strong conformism, people tend to stick to the existing norm, men conform to a traditional norm while female conform to an egalitarian norm, resulting in the mismatched norm equilibrium and low fertility. In the society with weak conformism, however, people easily update their norms corresponding to economic incentives. Thus the society would converge to shared care norms relatively faster than the society with strong conformism.

4 Equilibrium Selection: stochastic evolutionary model

To study equilibrium selection, I define a continuous time Markov process. Each agent possesses a random alarm clock with the same rate 1. The first time one of the clocks goes off, the agent possessing that clock receives the norm updating opportunity and the chosen agent update his or her norm according to a norm updating probability, which will be specified later. After the norm updating, agent picks his or her partner randomly from the opposite sex population and form a family.

Since only one agent is to revise his or her strategy at a time, the states change only by $\frac{1}{N}$. let $x^{\pm} = x \pm \frac{1}{N}$ and $y^{\pm} = y \pm \frac{1}{N}$ for shorthand notation to denote the changes in states. When an individual is chosen to update his or her norm given a state (x, y), there are four possible transitions from the current state to a different state: (x^+, y), (x^-, y), (x, y^+) and (x, y^-). For any two states $(x, y), (x, y)' \in S$ and $(x, y) \neq (x, y)'$, I assign a nonnegative number $\alpha((x, y), (x, y)')$ that denotes the rate at which the chain changes from the state (x, y) to the state $(x, y)'$ (Lawler, 2006). For example, $\alpha((x, y), (x^+, y))$ is the revision rate at which a male with E norm is chosen, and given the payoffs, he updates his norm from E

to T. The revision rates are as follows:

$$\begin{aligned}
\alpha((x,y),(x^+,y)) &= (1-x)\frac{\exp[\frac{1}{\eta}U(m,T,(x^+,y))]}{\exp[\frac{1}{\eta}U(m,E,(x,y))]+\exp[\frac{1}{\eta}U(m,T,(x^+,y))]} \\
\alpha((x,y),(x^-,y)) &= x\frac{\exp[\frac{1}{\eta}U(m,E,(x^-,y))]}{\exp[\frac{1}{\eta}U(m,T,(x,y))]+\exp[\frac{1}{\eta}U(m,E,(x^-,y))]} \\
\alpha((x,y),(x,y^+)) &= (1-y)\frac{\exp[\frac{1}{\eta}U(f,T,(x,y^+))]}{\exp[\frac{1}{\eta}U(f,E,(x,y))]+\exp[\frac{1}{\eta}U(f,T,(x,y^+))]} \\
\alpha((x,y),(x,y^-)) &= y\frac{\exp[\frac{1}{\eta}U(f,E,(x,y^-))]}{\exp[\frac{1}{\eta}U(f,T,(x,y))]+\exp[\frac{1}{\eta}U(f,E,(x,y^-))]}
\end{aligned} \tag{8}$$

The values inside the exponential function of each rate is the total payoff of individual i with his or her norm given a state. Thus the revision rate increases in the payoff of the target norm to which the revising-agent changes his or her current norm. For the revision rate in (8) I use so-called 'clever payoff evaluation' rule (Sandholm, 1998). It means that each agent compares the payoff of the current strategy in the current state and the payoff of the target strategy in the future state in which the agent plays his or her target strategy. The parameter $\eta \geq 0$ is the noise parameter representing the degree of noise. When $\eta \to \infty$, the terms in the revision rates in (8) approach $\frac{1}{2}$, implying that the strategy revising individuals randomize between strategies ignoring the payoffs. This case represents the situation where the observations of payoffs are too noisy, so the individual decision is highly perturbed by a noise. If $\eta \to 0$, then the terms in revision rates assume value 1 if and only if the utility of the target strategy is higher than the utility of the current strategy. In other words, the strategy revising individual surely chooses the best response at a given state, and this situation can be regarded as a situation where highly rational behaviors pervade with no perturbation. For this reason the equation (8) is called perturbed best response rule and the parameter η captures the degree of noise in the system.

In stochastic evolutionary game theory, the potential functions are frequently adopted in finding the explicit expressions for the stationary distribution. I first define a function V, called a potential function:

$$\begin{aligned}
V(x,y) &= N[(g)xy+(g-2n+w)x(1-y)+(w-n)(1-x)(1-y)] \\
&\quad +\frac{N}{2}[x^2\sigma+y^2\sigma+(1-x)^2\sigma+(1-y)^2\sigma]
\end{aligned} \tag{9}$$

The first term comes from within family payoffs depending on the spouse's norm and the second term from the conformist payoffs.

Since the state space is finite and the chain is irreducible (any state can be reached by any other state), the system admits a unique stationary distribution (Lawler, 2006). Let L be the infinitesimal generator of the chain specifying rates at which the chain jumps from a current state to a new state:$(L)_{((x,y),(x,y)')} = \alpha((x,y),(x,y)')$ if $(x,y) \neq (x,y)'$ and $(L)_{((x,y),(x,y))} = -\sum_{(x,y)' \in S} \alpha((x,y),(x,y)')$. Then the stationary distribution is defined as follows:

$$\mu(\{x,y\}) = \frac{\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}{\sum_{(x,y)\in S}\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]} \tag{10}$$

Then the condition for detailed balances needs to be checked, i.e.

$$\begin{aligned} & \mu((x,y))\alpha((x,y),(x,y)') \\ = \; & \mu((x,y)')\alpha((x,y)',(x,y)), \text{ for all } (x,y) \text{ and } (x,y)' \end{aligned}$$

There are only four possible cases to check.

$$\frac{\alpha((x,y),(x^{\pm},y))}{\alpha((x^{\pm},y),(x,y))} = \frac{\mu(\{x^{\pm},y\})}{\mu(\{x,y\})}, \quad \frac{\alpha((x,y),(x,y^{\pm}))}{\alpha((x,y^{\pm}),(x,y))} = \frac{\mu(\{x,y^{\pm}\})}{\mu(\{x,y\})}$$

By symmetry, it is enough to show for the case of (x^+,y) and (x,y^+), which is shown in the Appendix.

Proposition 3 *(Stationary distribution) The stationary distribution for the Markov chain defined by L is given by (10).*

Proof. See Appendix. ■

Young (1998) studies a given Markov process and its perturbed Markov processes by adding small errors, and defines a stochastically stable state as a state with a positive probability when the perturbation vanishes. In our current setting the perturbed processes in Young's specification correspond to the class of stochastic processes defined by L parameterized by η, and the unperturbed process can be viewed as the best response dynamics obtained by $\eta \to 0$. The Stochastically Stable State (SSS), thus, concerns only the state when η vanishes. Compared to this, the advantage of the explicit expression for the stationary distribution (10) is that it allows us to study the case of non-vanishing η as well as the limit of vanishing η, and provides more information about the long-run property of the system than SSS does.

The state (x,y) which maximizes the potential function $V(x,y)$ is the SSS of the game since as $\eta \to 0$ all weights will be put on this state. From the shape of potential function V, it is easy to see that local maximum only occurs at the vertices, $(1,1),(1,0),(0,1)$, and

$(0,0)$. Thus SSS is where the potential function, V, is maximized. Plugging these four states into (9) gives

$$\begin{aligned} V(1,1) &= Ng + N\sigma \\ V(0,0) &= N(w-n) + N\sigma \\ V(1,0) &= N(g-2n+w) + N\sigma \\ V(0,1) &= N\sigma \end{aligned} \tag{11}$$

First, it is obvious that the state $(0,1)$ and $(1,0)$ cannot be SSS. Because $g > 0$ and $n > g$, $V(0,1) < V(1,1)$ and $V(1,0) < V(0,0)$. State (1,1) or (0,0) will be a SSS. When $w < n + g$, the value of the potential function at the state of all male and female adopting T norm is higher than the value of the potential function at the state of all male and female adopting E norm, i.e. $V(1,1) > V(0,0)$, from the equation (11). Thus, the following results hold.

$$\begin{aligned} (1)\ (x,y) &= (1,1) \text{ is SSS if } w < n+g \\ (2)\ (x,y) &= (0,0) \text{ is SSS if } w > n+g \end{aligned}$$

Wive's wage (w), the utility from children (n) and husbands' cost of childrearing (g) determines SSS. Proposition 4 reports the results regarding norms and fertility.

Proposition 4 *Depending on women's wage and the cost of childrearing, there are following fertility regimes as a result of the evolution of gender norms.*

- *If women's wage is sufficiently low such that $w < n + g$, all males and females adopt a separate spheres norm and high fertility is likely to be observed.*
- *If women's wage is sufficiently high such that $w > n + g$, all males and females adopt a shared care norm and intermediate level of fertility is likely to be observed.*

In the previous section we have seen that there are different ESSs corresponding to women's market wage. The case (i) and (iii) have only one ESS, so equilibrium selection is not required. But the case (ii) in which $n < w < 2n$ has two ESSs, and the long run equilibrium can be selected using the result of Proposition 4. If $n < w < n + g$, separate spheres norm will be selected, but if $n + g < w < 2n$, shared care norm will be selected.

The equation (11) shows that the conformist component in the potential function, $N\sigma$, appears in all four states, implying that conformism does not alter the equilibrium selection.

Equilibrium selection process totally depends on the underlying payoffs within families. The reason is because I assume that the degree of conformism toward both norms are equal. Since conformist behavior is adopting a frequent norm in a population, it is reasonable to assume unbiased degree of conformism. However, it can be easily done to make the degree of conformism to vary by gender. Let σ_m and σ_f be the degree of conformism by male and female respectively. Then the potential function in (9) becomes

$$\begin{aligned} V(x,y) &= N[(g)xy + (g-2n+w)x(1-y) + (w-n)(1-x)(1-y)] \\ &\quad + \frac{N}{2}[x^2\sigma_m + y^2\sigma_f + (1-x)^2\sigma_m + (1-y)^2\sigma_f] \end{aligned}$$

And the corresponding potential of four states are

$$\begin{aligned} V(1,1) &= Ng + \frac{N}{2}(\sigma_m + \sigma_f) \\ V(0,0) &= N(w-n) + \frac{N}{2}(\sigma_m + \sigma_f) \\ V(1,0) &= N(g-2n+w) + \frac{N}{2}(\sigma_m + \sigma_f) \\ V(0,1) &= \frac{N}{2}(\sigma_m + \sigma_f) \end{aligned}$$

Similar to the unbiased degree of conformism, the conformist component in the potential function, $\frac{N}{2}(\sigma_m + \sigma_f)$, appears in all four states. Even though different degree of conformism by gender is allowed, it does not alter the SSS chosen in the case without conformism. According to the model, the mismatched norm state may be an absorbing state for a while, but as long as women's market work is compensated sufficiently high, we will observe intermediate fertility in the long-run.

5 Conclusion

I explore the evolution of gender norms and fertility regime in the presence of conformism under various economic environments. It shows how conformism alters the equilibrium. For example, even if families with both spouses having shared care norms receive higher within-family payoffs than families with mismatched norms (male with T norm and female with E norm), if individuals obtain strong conformist payoffs, the state of mismatched norms might be an ESS or an absorbing state in the evolutionary process.

In this paper I confine the investigation to the evolution of gender norms and fertility, but the results holds for any asymmetric game that admits potential functions. According to the

choice rule, I adopted perturbed best response. Perturbed best response is such that when an individual is chosen to revise his strategy, he/she chooses the strategy with the highest payoff among all possible strategies with some degrees of mistakes. Specifically I adopt the 'logit choice rule'(McKelvey and Palfrey, 1995; Young, 1998). In this case one could choose any strategies even though they have disappeared in the system, so the stochastic process is irreducible (every state can be reached from an arbitrary state by the evolution of time), and admits a unique stationary distribution. The advantage of logit choice rule is that under this strategy revision rule, the unique stationary distribution can be computed explicitly, thus one can easily study the long run property of the system, e.g. stochastic stabilities, by analyzing the expression of the stationary distribution. By contrast the imitation rule is such that upon revision an agent compares his/her payoff of current strategy to the payoff of a matched agent, and he/she imitates the other agent's strategy only when the other's payoff is higher (Weibull, 1995). Under this dynamic if some strategies disappear, they do not reappear in the system; thus the system is reducible and admits multiple absorbing states. Accommodating an imitation rule to this model would be an interesting extension.

References

Akerlof, G. A. and R. E. Kranton (2000). Economics and identity. *Quarterly Journal of Economics 105(3)*, 715–753.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal 75*, 493–517.

Bowles, S. (2004). *Microeconomics: Behavior, Institutions, and Evolution.* Princeton: Princeton Univ. Press.

Boyd, R. and P. J. Richerson (1985). *Culture and the Evolutionary Process.* University of Chicago Press.

Braunstein, E. and N. Folbre (2001). To honor and obey: Efficiency, inequality, and patriarchal property rights. *Feminist Economics 7*(1), 25–44.

Edlund, L. (1999). Son preference, sex ratios, and marriage patterns. *Journal of Political Economy 107*, 1275–1304.

Feyrer, J., B. Sacerdote, and A. S. Dora (2008). Will the stork return to europe and japan? understanding fertility within developed nations. *Journal of Economic Perspectives 22(3)*, 3–22.

Folbre, N. (2006). Chicks, hawks, and patriarchal institutions. In M. Altman (Ed.), *Handbook of Behavioral Economics*, pp. 499–516. Armong, NY: ME Sharpe.

Geddes, R. and D. Lueck (2002). The gains from self-ownership and the expansion of women's rights. *American Economic Review 92*(4), 1079–1092.

Hamilton, W. (1964). The genetical evolution of social behavior II. *Journal of Theoretical Biology 7*(1), 17–52.

Iversen, T. and F. Rosenbluth (2006). The political economy of patriarchy: How bargaining power shapes social norms and political attitudes. mimeo.

Laat, J. D. and A. S. Sanz (2006). Working women, men's home time and lowest low fertility. *ISER Working Paper 2006-23 Institute for Social and Economic Research*, University of Essex.

Lawler, G. F. (2006). *Introduction to Stochastic Process* (Second ed.). New York: Chapman and Hall/CRC.

McKelvey, R. D. and T. R. Palfrey (1995). Quantal response equilibria for normal form games. *Games and Economic Behavior 10*, 6–38.

Mills, M., L. Mencarini, M. L. Tanturri, and K. Begall (2008). Gender equity and fertily intentions in italy and the netherlands. *Demographic Research 18(1)*, 1–26.

Mincer, J. (1963). Market prices, opportunity costs, and income effects. In e. a. C. Christ (Ed.), *Measurement in economics: studies in mathematical economics and econometrics in memory of Yehuda Grunfeld*. Stanford: Stanford University Press.

Sandholm, W. H. (1998). Simple and clever decision rules for a model of evolution. *Economic Letters 61(2)*, 165–170.

Trivers, R. (1974). Parent-offspring conflict. *American Zoologist 14*, 249–264.

Weibull, J. (1995). *Evolutionary Game Theory*. Cambridge, MA: MIT Press.

Willis, R. J. (1973). A new approach to the economic theory of fertility behavior. *Journal of Political Economy 81(2)*, S14–S64.

Young, P. (1998). *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*. Princeton: Princeton Univ. Press.

Appendix

First I check the reversibility between the states (x, y) and (x^+, y). I find the ratio of the revision rates from (4) and (8) as follows:

$$\begin{aligned}
\frac{\alpha((x,y),(x^+,y))}{\alpha((x^+,y),(x,y))} &= \frac{1-x}{x^+}\frac{\frac{\exp[\beta U(m,T,(x^+,y))]}{\exp[\beta U(m,E,(x,y))]+\exp[\beta U(m,T,(x^+,y))]}}{\frac{\exp[\beta U(m,E,(x,y))]}{\exp[\beta U(m,T,(x^+,y))]+\exp[\beta U(m,E,(x,y))]}} \\
&= \frac{1-x}{x^+}\frac{\exp[\beta U(m,T,(x^+,y))]}{\exp[\beta U(m,E,(x,y))]} \\
&= \frac{1-x}{x^+}\exp[\beta\{U(m,T,(x^+,y)) - U(m,E,(x,y))\}] \\
&= \frac{1-x}{x^+}\exp[\beta\{3ny + n(1-y) + x^+\sigma \\
&\quad -(3n-g)y - (2n-g)(1-y) - (1-x)\sigma\}] \\
&= \frac{1-x}{x^+}\exp[\beta\{gy - (n-g)(1-y) + 2\sigma x + (\frac{1-N}{N})\sigma\}]
\end{aligned}$$

Then I find the ratio of stationary distributions from (10);

$$\begin{aligned}
\frac{\mu(\{x^+,y\})}{\mu(\{x,y\})} &= \frac{\frac{\binom{N}{Nx^+}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x^+,y)]}{\sum_{(x,y)\in S}\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}}{\frac{\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}{\sum_{(x,y)\in S}\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}} \\
&= \exp[\beta\{V(x^+,y) - V(x,y)\}]\frac{\binom{N}{Nx^+}\binom{N}{Ny}}{\binom{N}{Nx}\binom{N}{Ny}}
\end{aligned}$$

I show the result of the ratio of the reference distributions first.

$$\begin{aligned}
\frac{\binom{N}{Nx^+}\binom{N}{Ny}}{\binom{N}{Nx}\binom{N}{Ny}} &= \frac{\frac{N!}{(Nx^+)!(N-Nx^+)!}}{\frac{N!}{Nx!(N-Nx)!}} \\
&= \frac{1-x}{x^+}
\end{aligned}$$

Using the potential function (9), the term $V(x^+, y) - V(x, y)$ can be calculated as follows:

$$\begin{aligned}
& V(x^+, y) - V(x, y) \\
= \; & N[gx^+y + (g - 2n + w)x^+(1 - y) + (w - n)(1 - x^+)(1 - y) \\
& -gxy - (g - 2n + w)x(1 - y) - (w - n)(1 - x)(1 - y)] \\
& +\frac{N}{2}[(x^+)^2\sigma + y^2\sigma + (1 - x^+)^2\sigma + (1 - y)^2\sigma \\
& -x^2\sigma - y^2\sigma - (1 - x)^2\sigma - (1 - y)^2\sigma] \\
= \; & gy - (n - g)(1 - y) + 2\sigma x + (\frac{1 - N}{N})\sigma
\end{aligned}$$

From the two results above, I have

$$\frac{\mu(\{x^+, y\})}{\mu(\{x, y\})} = \frac{1 - x}{x^+} \exp[\beta N(gy - (n - g)(1 - y) + 2\sigma x)]$$

Similarly, I check the reversibility between (x, y) and (x, y^+). I find the ratio of the revision rates from (4) and (8) as follows:

$$\begin{aligned}
\frac{\alpha((x, y), (x, y^+))}{\alpha((x, y^+), (x, y))} &= \frac{1 - y}{y^+} \frac{\frac{\exp[\beta U(f,T,(x,y^+))]}{\exp[\beta U(f,E,(x,y))]+\exp[\beta U(f,T,(x,y^+))]}}{\frac{\exp[\beta U(f,E,(x,y))]}{\exp[\beta U(f,T,(x,y^+))]+\exp[\beta U(f,E,(x,y))]}} \\
&= \frac{1 - y}{y^+} \frac{\exp[\beta U(f, T, (x, y^+))]}{\exp[\beta U(f, E, (x, y))]} \\
&= \frac{1 - y}{y^+} \exp[\beta\{U(f, T, (x, y^+)) - U(f, E, (x, y))\}] \\
&= \frac{1 - y}{y^+} \exp[\beta\{nx + n - w + 2\sigma y + (\frac{1 - N}{N})\sigma\}]
\end{aligned}$$

Then I find the ratio of stationary distributions from (**??**) and (10) :

$$\begin{aligned}
\frac{\mu(\{x, y^+\})}{\mu(\{x, y\})} &= \frac{\frac{\binom{N}{Nx}\binom{N}{Ny^+}\exp[\frac{1}{\eta}V(x,y^+)]}{\sum_{(x,y)\in S}\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}}{\frac{\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}{\sum_{(x,y)\in S}\binom{N}{Nx}\binom{N}{Ny}\exp[\frac{1}{\eta}V(x,y)]}} \\
&= \exp[\beta\{V(x, y^+) - V(x, y)\}]\frac{\binom{N}{Nx}\binom{N}{Ny^+}}{\binom{N}{Nx}\binom{N}{Ny}}
\end{aligned}$$

Then, by the similar calculation, I have

$$\begin{aligned}
\frac{\binom{N}{Nx}\binom{N}{Ny^+}}{\binom{N}{Nx}\binom{N}{Ny}} &= \frac{1-y}{y^+} \\
V(x,y^+) - V(x,y) &= nx + n - w + 2\sigma y + (\frac{1-N}{N})\sigma
\end{aligned}$$

Thus, I have the following result.

$$\frac{\mu(\{x,y^+\})}{\mu(\{x,y\})} = \frac{1-y}{y^+}\exp[\beta\{nx + n - w + 2\sigma y + (\frac{1-N}{N})\sigma\}]$$

I verify that the distribution (9) satisfies the detailed balances.